Identically closed two-form for covariant phase space quantization of Dirac-Nambu-Goto p-branes in a curved spacetime

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Using a fully covariant formalism given by Carter for the deformation dynamics of p-branes governed by the Dirac-Nambu-Goto action in a curved background, it is proved that the corresponding Witten's phase space is endowed with a covariant symplectic structure, which can serve as a starting point for a phase space quantization of such objects. Some open questions for further research are outlined.

Keywords: Symplectic structure, canonical quantization, p-branes.

I. INTRODUCTION

A p-dimensional brane is a relativistic extended object propagating in a background spacetime manifold of some dimension N. Such a dynamical system is defined in terms of fields with support confined to a (p+1)-dimensional world sheet manifold that the p-brane swept out in the course of its evolution, in such a way that $N \geq (p+1)$. In this sense, a 0-brane is a point particle, a 1-brane is a string, a 2-brane a domain wall or membrane, and so on. The extreme case (N-1)-brane corresponds to that of a continuous medium for which the confinement condition is redundant. It is well known the very wide range of physically diverse phenomena modeled as a p-brane of an appropriate dimension. Originally, Dirac proposed his bubble-membrane as "an extensible model of the electron" [1], and Nambu and Goto modeled hadronic matter using a relativistic string [2], and later string theory was developed as a possible theory of strong interactions [3]. Subsequently, in the last years the theory of extended objects is considered the point of departure in the construction of the modern string/M-theory, which claims to give a consistent unified description of all interactions including gravity, and particularly a way of overcoming the non-renormalizability of point-like theories.

In the last years a considerable amount of effort has been devoted for developing a quantum field theory of extended objects (which in fact, will constitute the ultimate framework for a complete string/M-theory), however it has not yet been fully developed. The quantization of extended objects is a very complicated problem in physics because, among other things, the theory is highly non-linear and the standard methods are not directly applied, even for extended objects of simple topologies. In this manner, the main purpose of the present work is to explore the basic elements of one of such methods, the so called phase space quantization that was originally introduced by Witten [4, 5], and that is based on a covariant description of the standard canonical formalism. We shall restrict the present analysis to the bosonic p-branes of the simplest topology, the internally structureless p-dimensional generalization of the Dirac-Nambu-Goto (DNG) membranes embedded in a curved spacetime, for clarifying our basic ideas and to prepare the background for a future treatment of physically more interesting cases.

In the next section, we summarize the DNG action for p-branes and the covariant description of the corresponding deformations given by Carter [6], which is essential for our present aims. In Section III, using the concept of (self-)adjoint operators, we derive a local continuity equation that will allow to identify a bilinear form on deformations of classical solutions, which will be connected directly to the wanted canonical structure on the phase space; the later will be explained in Section IV. In Section V the closeness of the bilinear form previously constructed is proved. We finish in Section VI with some discussions about our results and future extensions.

II. THE DNG ACTION AND FIRST ORDER DEFORMATIONS

The DNG action for p-branes in an arbitrary curved background is given by an integral proportional to the area swept out by the world sheet:

$$\mathcal{S} = \int \widetilde{L} d\widetilde{\Sigma}, \quad \widetilde{L} = -m^{\rho+1}, \tag{1}$$

where m is a fixed parameter, and $d\widetilde{\Sigma}$ is the surface measure element induced on the world sheet by the background metric ($g_{\mu\nu}$). Using the decomposition $g^{\mu\nu} = \eta^{\mu\nu} + \perp^{\mu\nu}$ of the background metric in terms of the tangential and orthogonal projectors to the world sheet, with the properties

$$\eta^{\mu}{}_{\rho}\eta^{\rho}{}_{\nu} = \eta^{\mu}{}_{\nu}, \quad \perp^{\mu}{}_{\rho}\perp^{\rho}{}_{\nu} = \perp^{\mu}{}_{\nu}, \quad \eta^{\mu}{}_{\rho}\perp^{\rho}{}_{\nu} = 0, \tag{2}$$

the dynamical equations of motion of the action (1) are expressible as the harmonicity condition

$$\widetilde{\nabla}_{\mu}\eta^{\mu}{}_{\nu} = 0, \tag{3}$$

where $\widetilde{\nabla}_{\mu} \equiv \eta^{\nu}{}_{\mu}\nabla_{\nu}$ is the tangential covariant differentiation (∇_{ν} denotes Riemannian covariant differentiation with respect to the background metric), which is the only meaningful for tensor fields whose support is confined to the world sheet.

According to the Carter approach [6], the linearized dynamics of first order deformations of the p-branes under consideration, can be described in terms of the infinitesimal displacement vector field ξ^{μ} by the equations:

$$(\perp^{\mu}{}_{\lambda}\widetilde{\nabla}_{\nu}\widetilde{\nabla}^{\nu} - 2K_{\lambda}{}^{\nu\mu}\widetilde{\nabla}_{\nu} + \perp^{\mu}{}_{\nu}\eta^{\rho\sigma}\mathcal{R}_{\rho}{}^{\nu}{}_{\sigma\lambda})\xi^{\lambda} \equiv (\mathcal{P}\xi^{\lambda})^{\mu} = 0, \tag{4}$$

where $K_{\mu\nu}{}^{\rho} = \eta^{\lambda}{}_{\nu}\widetilde{\nabla}_{\mu}\eta^{\rho}{}_{\lambda}$ is the *second fundamental tensor*, and $\mathcal{R}_{\mu\rho\nu\sigma}$ denotes the Riemann curvature of the background spacetime. Note that the linear operator $\mathcal{P}$ is taking vector fields into themselves. ξ^{μ} can be identified with the corresponding first order change in the coordinate fields x^{μ} of the background, $\xi^{\mu} = \delta_L x^{\mu}$, where δ_L denotes a *Lagrangian* deformation in the sense of being defined with respect to a reference system comoving with the relevant displacement [6]. For fields with support confined to the world sheet, their Lagragian deformation reduces to the Lie differentiation with respect to the displacement vector $\xi^{\mu}(\mathcal{L}_{\xi})$. For fields whose support is not confined to the world sheet, there will be a *fixed point* differential (δ_E) in such a way that $\delta_L = \mathcal{L}_{\xi} + \delta_E$. In particular, in the Carter approach is considered that *the background metric is fixed*, $\delta_E g_{\mu\nu} = 0$, and then

$$\delta_L g_{\mu\nu} = \mathcal{L}_{\xi} g_{\mu\nu} = \nabla_{\mu}\xi_{\nu} + \nabla_{\nu}\xi_{\mu}, \quad \delta_L g^{\mu\nu} = -(\nabla^{\mu}\xi^{\nu} + \nabla^{\nu}\xi^{\mu}), \tag{5}$$

which will be useful below. For more details about this section, see directly Ref. [6] (and also [7], particularly the sections 4.1–4.4).

III. ADJOINT OPERATORS AND COVARIANTLY CONSERVED CURRENTS

In this Section we shall construct from Eq. (4), and using the concept of self-adjoint operators, a bilinear form on the displacement vector ξ^μ and their derivatives, which will be world sheet covariantly conserved.

3.1. Self-adjoint operators and local continuity laws

The general relationship between adjoint operators and covariantly conserved currents has been already given in previous works (see for example [8] and references cited therein), however we shall discuss it in this section for completeness.

If $\mathcal{P}$ is a linear partial differential operator which takes matrix-valued tensor fields into themselves, then, the adjoint operator of $\mathcal{P}$, is that operator $\mathcal{P}^\dagger$, such that

$$f^{\rho\sigma\cdots}[\mathcal{P}(g_{\mu\nu\ldots})]_{\rho\sigma\ldots} - [\mathcal{P}^\dagger(f^{\rho\sigma\cdots})]^{\mu\nu\cdots}g_{\mu\nu\ldots} = \nabla_\mu \mathcal{J}^\mu, \tag{6}$$

where $\mathcal{J}^\mu$ is some vector field. From this definition, if $\mathcal{Q}$ and $\mathcal{R}$ are any two linear operators, one easily finds the following properties:

$$(\mathcal{Q}\mathcal{R})^\dagger = \mathcal{R}^\dagger\mathcal{Q}^\dagger, \qquad (\mathcal{Q}+\mathcal{R})^\dagger = \mathcal{Q}^\dagger + \mathcal{R}^\dagger,$$

and in the case of a function F,

$$F^\dagger = F,$$

which will be used implicitly below.

From Eq. (6) we can see that this definition automatically guarantees that, if $\mathcal{P}$ is a self-adjoint operator ($\mathcal{P}^\dagger = \mathcal{P}$), and the fields f and g correspond to a pair of solutions admitted by the linear system $\mathcal{P}(f) = 0 = \mathcal{P}(g)$, then we obtain the continuity law $\nabla_\mu \mathcal{J}^\mu = 0$, which establishes that $\mathcal{J}^\mu$ is a covariantly conserved current, bilinear on the fields f and g. This fact means that for any self-adjoint homogeneous equation system, one can always to construct a conserved current. Although this result has been established assuming only tensor fields and the presence of a single equation, such a result can be extended in a direct way to equations involving spinor fields, matrix fields, and the presence of more that one field.

3.2. Self-adjointness of the linearized dynamics

In this section we shall demonstrate that the operator $\mathcal{P}$ in Eq. (4) is indeed self-adjoint. With this purpose, let $\acute{\xi}_\mu$ and ξ_λ be two vector fields (which will be identified as a pair of solutions of Eq. (4)), and following the definition (6), we need to analyze the scalar expression $\acute{\xi}_\mu(\mathcal{P}\xi^\lambda)^\mu$, and thus the following identities will be useful (and very easy to verify):

$$\acute{\xi}_\mu \perp^\mu_\lambda \widetilde{\nabla}_\nu \widetilde{\nabla}^\nu \xi^\lambda \equiv \widetilde{\nabla}_\nu \mathcal{J}_1^\nu + \{[\perp^\mu_\lambda \widetilde{\nabla}_\nu \widetilde{\nabla}^\nu + 2(\widetilde{\nabla}^\nu \perp^\mu_\lambda)\widetilde{\nabla}_\nu + (\widetilde{\nabla}_\nu \widetilde{\nabla}^\nu \perp^\mu_\lambda)]\acute{\xi}_\mu\}\xi^\lambda, \tag{7}$$

$$-2\acute{\xi}_\mu K_\lambda{}^{\nu\mu}\widetilde{\nabla}_\nu \xi^\lambda \equiv \widetilde{\nabla}_\nu \mathcal{J}_2^\nu + 2\{[K_\lambda{}^{\nu\mu}\widetilde{\nabla}_\nu + (\widetilde{\nabla}_\nu K_\lambda{}^{\nu\mu})]\acute{\xi}_\mu\}\xi^\lambda, \tag{8}$$

where $\mathcal{J}_1^\nu = \perp^\mu_\lambda \acute{\xi}_\mu \widetilde{\nabla}^\nu \xi^\lambda - \widetilde{\nabla}^\nu(\perp^\mu_\lambda \acute{\xi}_\mu)\xi^\lambda$, and $\mathcal{J}_2^\nu = -2K_\lambda{}^{\nu\mu}\acute{\xi}_\mu \xi^\lambda$. The first term $\perp^\nu_\lambda \widetilde{\nabla}_\nu \widetilde{\nabla}^\nu \acute{\xi}_\mu$ in the {} terms in Eq. (7) is directly the first term of $(\mathcal{P}\acute{\xi}_\mu)_\lambda$. We add (and subtract) the complementary terms $(-2K^{\mu\nu\lambda}\widetilde{\nabla}_\nu + \perp^\lambda_\nu \eta^{\rho\sigma}\mathcal{R}_\rho{}^\nu{}_\sigma{}^\mu)\acute{\xi}_\mu$ for making $(\mathcal{P}\xi^{'}_\mu)_\lambda$ on the right hand-side of Eq. (7) (following Eq. (4)), and to obtain, after some arrangements, that

$$\begin{aligned}\acute{\xi}_\mu(\mathcal{P}\xi^\lambda)^\mu &= (\mathcal{P}\acute{\xi}_\mu)_\lambda \xi^\lambda + \widetilde{\nabla}_\nu(\mathcal{J}_1^\nu + \mathcal{J}_2^\nu) + 2[K^{\mu\nu}{}_\lambda + K_\lambda{}^{\nu\mu} + \widetilde{\nabla}^\nu \perp^\mu_\lambda](\widetilde{\nabla}_\nu \acute{\xi}_\mu)\xi^\lambda \\ &\quad + (\widetilde{\nabla}_\nu \widetilde{\nabla}^\nu \perp^\mu_\lambda + 2\widetilde{\nabla}_\nu K_\lambda{}^{\nu\mu})\acute{\xi}_\mu \xi^\lambda + \eta^{\rho\sigma}(\perp^\mu_\nu \mathcal{R}_\rho{}^\nu{}_{\sigma\lambda} - \perp_{\lambda\nu}\mathcal{R}_\rho{}^\nu{}_\sigma{}^\mu)\acute{\xi}_\mu \xi^\lambda,\end{aligned} \tag{9}$$

where it remains to demonstrate that, in according to Eq. (6), if $\mathcal{P}$ is self-adjoint, then the last three terms in Eq. (9) correspond at most to a pure divergence, such as the second term. This is effectively the case; using the definition of $K^{\mu\nu}{}_\lambda$ and the first of properties (2), it is very easy to find that $K^{\mu\nu}{}_\lambda + K_\lambda{}^{\nu\mu} + \widetilde{\nabla}^\nu \perp^\mu_\lambda = \widetilde{\nabla}^\nu \eta^\mu_\lambda + \widetilde{\nabla}^\nu \perp^\mu_\lambda = 0$, and thus the third term in Eq. (9) vanishes. Furthermore, using the definition $2\nabla_{[\mu}\nabla_{\nu]}\mathcal{A}_\rho = \mathcal{R}_{\mu\nu\rho}{}^\lambda \mathcal{A}_\lambda$ and the relation $\perp^\mu_\nu = g^\mu_\nu - \eta^\mu_\nu$, one can demonstrate easily that

$$\eta^{\sigma\rho}(\perp^\mu_\nu \mathcal{R}_\rho{}^\nu{}_{\sigma\lambda} - \perp_{\lambda\nu}\mathcal{R}_\rho{}^\nu{}_\sigma{}^\mu)\acute{\xi}_\mu \xi^\lambda = \widetilde{\nabla}_\nu \mathcal{J}_3^\nu + (\widetilde{\nabla}^\nu \perp^\mu_\lambda + 2K_\lambda{}^{\nu\mu})\widetilde{\nabla}_\nu(\acute{\xi}_\mu \xi^\lambda), \tag{10}$$

where $\mathcal{J}_3^\nu = \widetilde{\nabla}_\mu(\acute{\xi}^\nu \xi^\mu - \acute{\xi}^\mu \xi^\nu)$. Hence, from Eqs. (9) and (10), we have finally

$$\acute{\xi}_\mu(\mathcal{P}\xi^\lambda)^\mu = (\mathcal{P}\acute{\xi}_\mu)_\lambda \xi^\lambda + \widetilde{\nabla}_\nu \widetilde{\mathcal{J}}^\nu, \tag{11}$$

where $\widetilde{\mathcal{J}}^\nu = \mathcal{J}_1^\nu + \mathcal{J}_2^\nu + \mathcal{J}_3^\nu + (\widetilde{\nabla}^\nu \perp^\mu_\lambda + 2K_\lambda{}^{\nu\mu})\acute{\xi}_\mu \xi^\lambda$, which has a remarkable simplification by substituting the explicit forms of $\mathcal{J}_1$, $\mathcal{J}_2$, and $\mathcal{J}_3$:

$$\widetilde{\mathcal{J}}^\nu = (\eta^{\mu\nu}\perp_{\sigma\rho} + 2\eta^\nu{}_{[\sigma}\eta_{\rho]}{}^\mu)[\acute{\xi}^\sigma \widetilde{\nabla}_\mu \xi^\rho - (\widetilde{\nabla}_\mu \acute{\xi}^\rho)\xi^\sigma]. \tag{12}$$

Therefore, Eq. (11) has the form of (6) with $\mathcal{P}^\dagger = \mathcal{P}$ and $\widetilde{\mathcal{J}}^\nu$ will be world sheet covariantly conserved

$$\widetilde{\nabla}_\nu \widetilde{\mathcal{J}}^\nu = 0, \tag{13}$$

if $\acute{\xi}_\mu$ and ξ_λ correspond to a pair of solutions of the linearized dynamics: $(\mathcal{P}\xi^\lambda)_\mu = 0 = (\mathcal{P}\acute{\xi}_\mu)_\lambda$. The current $\widetilde{\mathcal{J}}^\nu$ obtained in (12) is essentially that obtained by Carter [6] directly from the second order variation theory and it is understood as a conventional symplectic Noetherian current on the world sheet. Hence, we have up to here only reproduced by another way, part of the results obtained in [6]. However, the main contribution of the present work is contained from the next section, where we shall give, following Witten [4, 5], a nonconventional physical meaning to $\widetilde{\mathcal{J}}^\nu$ on the covariant phase space, which will lead finally to the construction of a symplectic structure for the theory. For simplifying our calculations, we shall work in the orthogonal gauge

$$\eta^\mu{}_\nu \xi^\nu = 0, \tag{14}$$

which removes the nonphysically observable tangential projection of the deformation; in this gauge $\widetilde{\mathcal{J}}^\nu$ has an additional simplification [6]:

$$\widetilde{\mathcal{J}}^\nu = \acute{\xi}^\rho \widetilde{\nabla}^\nu \xi_\rho - (\widetilde{\nabla}^\nu \acute{\xi}^\rho)\xi_\rho. \tag{15}$$

IV. WITTEN PHASE SPACE

In according to Witten [4, 5], in a given physical theory, *the classical phase space is the space of solutions of the classical equations of motion*, which corresponds to a manifestly covariant definition. Based on this definition, the idea of giving a covariant description of the canonical formalism consists in describing Poisson brackets of the theory in terms of a symplectic structure on such a phase space in a covariant way, instead of choosing p's and q's. Strictly speaking, a symplectic structure is a (non degenerate) closed two-form on the phase space; hence, for working in this scheme an exterior calculus associated with the phase space is fundamental. We summarize and adjust all these basic ideas about phase space quantization given in Ref. [4, 5] for the case of p-branes treated here.

First, the phase space of DNG p-branes is the space of solutions of Eqs. (3), and we shall call it Z. Any (unperturbed) background quantity such as the background and internal metrics, the projection tensors, etc., will be associated with zero-forms on Z [4, 5]. The Lagrangian deformation δ_L acts as an exterior derivative on Z, taking k-forms into $(k+1)$-forms, and it should satisfy

$$\delta_L^2 = 0, \tag{16}$$

and the Leibniz rule

$$\delta_L(AB) = \delta_L AB + (-1)^A A\delta_L B.$$

In particular, $\xi^\mu = \delta_L x^\mu$ is the exterior derivative of the zero-form x^μ, and corresponds to an one-form on Z, and thus is an anticommutating object: $\xi^\mu\xi^\lambda = -\xi^\lambda\xi^\mu$. In according to (16), ξ^μ will be closed, $\delta_L\xi^\mu = \delta_L^2 x^\mu = 0$, which is evident from the explicit form of $\delta_L\xi^\mu$ given in [6]:

$$\delta_L\xi^\mu = -\Gamma^\mu_{\lambda\nu}\xi^\lambda\xi^\nu = 0, \tag{17}$$

which vanishes because of the symmetry of the background connection $\Gamma^\mu_{\lambda\nu}$ in its indices λ and ν and the anticommutativity of the ξ^λ on Z.

In this manner, $\widetilde{\mathcal{J}}^\nu$ in Eq. (15), being bilinear in the one-forms $\acute{\xi}^\rho$ and ξ_ρ, corresponds to a two-form on Z; since $\widetilde{\nabla}_\mu$ depends only an unperturbed background and world sheet quantities (zero-forms), $\widetilde{\nabla}_\mu\xi_\rho$ will be an one-form on Z, such as ξ_ρ. The bilinear product $\acute{\xi}^\rho\widetilde{\nabla}_\mu\xi_\rho$ must be understood strictly as a wedge product of one-forms on Z: $\acute{\xi}^\rho \wedge \widetilde{\nabla}_\mu\xi_\rho$; however, we avoid the explicit use of $\wedge$ (such as Ref. [4, 5]), taking into account always that for differential forms $AB = (-1)^{AB}BA$.

With these ideas, we can set $\acute{\xi}^\rho = \xi^\rho$ in Eq. (15), without loosing generality, and considering that $(\widetilde{\nabla}_\mu\xi^\rho)\xi_\rho = -\xi_\rho\widetilde{\nabla}_\mu\xi^\rho$, one obtains essentially the following two-form on Z:

$$\widetilde{\mathcal{J}}^\nu = \xi^\rho\widetilde{\nabla}^\nu\xi_\rho. \tag{18}$$

V. THE SYMPLECTIC STRUCTURE ON Z

As mentioned, a symplectic structure is a closed two-form on Z. The closeness means that the exterior derivative of such two-form vanishes on Z. In this section we shall construct, from the two-form (18), a symplectic structure for DNG p-branes.

Since the world sheet is an orientable manifold, integration is defined in straightforward way, and we can construct, following Witten [4, 5], the following two-form on Z:

$$\omega \equiv \int_{\Sigma} \sqrt{-\gamma}\widetilde{\mathcal{J}}^{\mu} d\widetilde{\Sigma}_{\mu}, \tag{19}$$

where γ is the determinant of the world sheet metric and Σ is a spacelike section of the world sheet manifold and corresponds to an initial value p-surface for the configuration of the p-brane; $d\widetilde{\Sigma}_{\mu}$ is the surface measure element of Σ, and is normal on Σ and tangent to the world sheet. Employing Green's theorem in an usual way (see for example Eq. (95) in [7]), the world sheet current conservation law (13) ensures that ω in (19) is independent on the choice of Σ:

$$\int_{\Sigma} \sqrt{-\gamma}\widetilde{\mathcal{J}}^{\mu} d\widetilde{\Sigma}_{\mu} = \int_{\Sigma'} \sqrt{-\gamma}\widetilde{\mathcal{J}}^{\mu} d\widetilde{\Sigma}'_{\mu}, \tag{20}$$

and is, in particular, Poincaré invariant. We shall demonstrate now that ω is indeed a closed two-form on Z. From Eq. (19), the exterior derivative of ω is given by

$$\delta_L \omega = \int_{\Sigma} [(\delta_L \sqrt{-\gamma})\widetilde{\mathcal{J}}^{\mu} + \sqrt{-\gamma}\delta_L \widetilde{\mathcal{J}}^{\mu}] d\widetilde{\Sigma}_{\mu}; \tag{21}$$

considering that $\delta_L \sqrt{-\gamma} = 0$ corresponds to the first order action variation (see Eqs. (1) and (3)), the closeness of ω holds if $\widetilde{\mathcal{J}}^{\mu}$ itself is closed. For demonstrating this property, we rewrite the current (18) as $\widetilde{\mathcal{J}}^{\mu} = \eta^{\mu\nu}\mathcal{J}_{\nu}$, where $\mathcal{J}_{\nu} \equiv \xi_{\rho}\nabla_{\nu}\xi^{\rho}$, and let us show first that $\mathcal{J}_{\nu}$ is an *exact* two-form on Z.

The Lagrangian variation of tensor fields in the Carter approach (and that corresponds to the exterior derivative of such fields on Z in the present approach), is defined in terms of the affinely parameterised geodesic equation

$$\xi^{\nu}\nabla_{\nu}\xi^{\mu} = 0, \tag{22}$$

which is fully equivalent to Eq. (134) in [7]; $\xi^{\mu} = \delta_L x^{\mu}$ is the affinely normalized tangent vector along the geodesic. In particular, the exterior derivative of the one-form $\xi_{\mu} \equiv g_{\mu\nu}\xi^{\nu}$ is given by

$$\delta_L \xi_{\mu} = (\delta_L g_{\mu\nu})\xi^{\nu} + g_{\mu\nu}\delta_L \xi^{\nu} = (\nabla_{\mu}\xi_{\nu} + \nabla_{\nu}\xi_{\mu})\xi^{\nu} = (\nabla_{\mu}\xi_{\nu})\xi^{\nu} = -\mathcal{J}_{\mu}, \tag{23}$$

where Eqs. (5), (17), and (22) have been used. Hence, Eq. (23) shows that $\mathcal{J}_{\mu}$ is an *exact* two-form (corresponds to the exterior derivative of an one-form), and automatically a closed two-form, in view of (16):

$$\delta_L \mathcal{J}_{\mu} = -\delta_L^2 \xi_{\mu} = 0. \tag{24}$$

In this manner $\widetilde{\mathcal{J}}^{\mu} = -\eta^{\mu\nu}\delta_L \xi_{\nu}$ on Z, and

$$\delta_L \widetilde{\mathcal{J}}^{\mu} = -(\delta_L \eta^{\mu\nu})\delta_L \xi_{\nu}, \tag{25}$$

in virtue of (24). However, from the property (16) and the Leibniz rule, Eq. (25) can be rewritten as

$$\delta_L \widetilde{\mathcal{J}}^{\mu} = \delta_L[(\delta_L \eta^{\mu\nu})\xi_{\nu}], \tag{26}$$

and considering that $\delta_L \eta^{\mu\nu} = -\eta^{\mu\rho}\eta^{\nu\sigma}\delta_L g_{\rho\sigma}$ [6, 7],

$$(\delta_L \eta^{\mu\nu})\xi_{\nu} = -\eta^{\mu\rho}\delta g_{\rho\sigma}(\eta^{\nu\sigma}\xi_{\nu}) = 0, \tag{27}$$

in according to the orthogonality condition (14). Thus, Eqs. (26) and (27) imply that $\delta_L \widetilde{\mathcal{J}}^{\mu} = 0$, and ω is identically closed on Z, in according to Eq. (21). Therefore, (19) is our wanted symplectic structure on Z for DNG p-branes.

It remains to discuss the gauge invariance properties of ω. Since all fields appearing in the definitions of ω transform homogeneously like tensors (which is precisely a virtue of the Carter formalism for deformations employed here), ω is invariant under spacetime diffeomorphisms. Similarly, ω involves integration of a world sheet scalar density, and then is invariant under world sheet reparametrizations. In this manner, our symplectic structure ω *inherits* the covariant properties of the deformation formalism, from which it has been constructed.

VII. REMARKS AND PROSPECTS

In this manner, we have obtained a symplectic structure for DGN p-branes propagating in an arbitrary curved background. Such a structure is covariant in the strong sense of being expressed in terms only of ordinary tensors, and not of internal world sheet degrees of freedom. It is important to emphasize that, in accordance with the above results, ω emerges in a natural way and without any restrictions or additional assumptions, which suggests a deeper research of the symplectic structure ω, with the idea that such a Hamiltonian scheme serves as the base of a covariant canonical quantization of such objects on a curved background. Specifically the issue of the degenerate directions and the existence of *global* symplectic potentials on the phase space of the theory has been considered in a recent paper [9], and additionally a *weakly* covariant description of the phase space for a restricted class of topological defects has been also given in [10].

It is opportune to mention that the restriction on the background metric considered in the present letter (see paragraph before Eq.(5)), has been dropped in a recent work by R. Battye and B. Carter [11], and works along these lines are in progress for overcoming such a limitation. Finally, physically more interesting cases will be treated elsewhere following the ideas presented here.

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